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06MAT21

Second Semester B.E. Degree Examination, December 2010
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: *1. Answer any FIVE full questions, choosing at least two from each part.*
2. Answer all objective type questions only in OMR sheet page 5 of the Answer Booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1 a. Select the correct answer in each of the following :

i) The radius of curvature of a curve y = c cosh (x/c) is

A) $\frac{c^2}{2}$ B) y^2/c C) $3c/8\sqrt{2}$ D) c

ii) The value of C of the Rolle's theorem for $f(x) = \frac{\text{Sin}2x}{e^{2x}}$ in [0, π/2] is

A) 3π B) π/8 C) π D) = π/5

iii) Maclaurins series expansion of e^{Sinx} is

A) $1 + x + \frac{x^2}{2} - \frac{x^4}{8} + ...$ B) $x - \frac{x^3}{3!} + \frac{x^5}{5!} + ...$

C) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + ...$ D) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} + ...$

iv) The value of C of the Lagrange's mean value theorem for f(x) = logx in [1, e] is

A) 1.2 B) 2.5 C) 1.7 D) 3.2 **(04 Marks)**

b. Find the radius of curvature for the curve, $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on it. **(04 Marks)**

c. State and prove Lagrange's mean value theorem. **(06 Marks)**

d. Expand log (secx) up to the term x^4 using Maclaurin's series. **(06 Marks)**

2 a. Select the correct answer in each of the following :

i) $\underset{x \to \pi/2}{\text{Lt}} \frac{\log(\text{Sinx})}{(\pi/2 - x)^2}$ equals

A) $\frac{1}{2}$ B) 2 C) π/2 D) - 1/2

ii) For finding extreme values of f(x, y),

A) $\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0$ B) $\frac{\partial^2 f}{\partial y \partial x} = 0$

C) $\frac{\partial^2 f}{\partial y^2} = 0$ D) None of these

iii) The rectangular box of maximum volume and a given surface area is

A) a triangle B) a rectangle C) a cube D) None of these

iv) For finding the stationary value of u (x, y, z) subject to the condition φ (x, y, z) = c, the relation is

A) F = u (x, y, z) + λ φ (x, y, z) = c B) F (x, y) = 0

C) $\frac{\partial f}{\partial x} = 0$ D) None of these **(04 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. Evaluate $\lim_{x \to 0} \tan x \log x$. **(04 Marks)**

c. Expand Sin (xy) about $(1, \pi/2)$ up to second degree terms. **(06 Marks)**

d. Find the extreme values of the function, $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$. **(06 Marks)**

3 a. Select the correct answer in each of the following :

i) $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy\, dx =$

A) 2/5 B) 3/35 C) 3/2 D) 5/2

ii) $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2) dz\, dy\, dx =$

A) $8abc(a^2 + b^2 + c^2)/3$ B) $\frac{8abc}{3}$

C) $9ab^2c$ D) $\frac{a^2bc^2}{3}$

iii) The value of $\beta(m, n)$ is

A) $\int_0^1 x^{m-1}(1-x)^{n-1} dx$ B) $\int_0^\infty e^{-x} x^{n-1} dx$

C) $\int_1^\infty e^{x^2+y^2} dx$ D) None of these

iv) The value of $\Gamma(1/2)$ is

A) $\sqrt{\pi}$ B) π C) $\pi^2/2$ D) $\pi/\sqrt{2}$. **(04 Marks)**

b. Evaluate $\iint xy(x + y)\, dy\, dx$ taken over the area between $y = x^2$ and $y = x$. **(04 Marks)**

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$. **(06 Marks)**

d. Prove that $\beta(m, n) = \beta(n, m)$. **(06 Marks)**

4 a. Select the correct answer in each of the following :

i) $\vec{F}$ is said to be irrotational, if

A) $\oint_c \vec{F}.d\vec{r} = 0$ B) $\vec{F} d\vec{r} = 0$ C) $\vec{F}.\vec{r} = 0$ D) None of these

ii) If $\vec{F}$ is the force acting upon a particle in displacing it along the curve c to the other end, then the total work done by $\vec{F}$ is.

A) $\int \vec{F} \times d\vec{r}$ B) $\int \vec{F} . d\vec{r}$ C) $\int d\vec{r}$ D) None of these

iii) Green's theorem in the plane is a special case of

A) Gauss theorem B) Euler's theorem

C) Stokes theorem D) Baye's theorem

iv) The cylindrical polar co-ordinates are (ρ, ϕ, z) given by

A) $x = \rho \cos\phi, y = \rho \sin\phi, z = z$ B) x, y, z

C) $x = \sin\theta, y = \cos\theta$ D) None of these **(04 Marks)**

b. If $\vec{F} = (3x^2 + 6y)\, i - 14\, yzj + 20\, xz^2k$, evaluate $\int \vec{F}.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve given by $x = t$, $y = t^2$ $z = t^3$. **(04 Marks)**

c. Verify Stokes theorem for $\vec{F} = (2x - y)\, i - yz^2 j - y^2 zk$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$, C is its boundary. **(06 Marks)**

d. If $\phi = xyz$, find $\nabla^2\phi$ in the cylindrical system. **(06 Marks)**

PART – B

5 a. Select the correct answer in each of the following :

i) P.I. of the differential equation $(D^2 + 4D + 4)\, y = e^{-2x}$ is

A) e^x B) $\frac{x^2 e^{-2x}}{2}$ C) x^3 D) $\frac{xe^{2x}}{3}$

ii) The solution of the differential equation $(D^2 + 2D + 1)\, y = 0$ is

A) $c_1e^x + c_2\, e^{-x}$ B) $(c_1 + c_2x)e^{-x}$

C) c_1e^x D) $c_1 + c_2\, e^{-2x}$

iii) The roots of the A. E. with differential equation $(D^3 - D^2 + 4D - 4)\, y = 0$ are

A) 1, ±2i B) 1, 2, 1 C) 1, 3, 2 D) 2, 2, 2

iv) The particular solution of the differential equation $f(D)y = e^{ax}$ is

A) $\frac{e^{ax}}{f(a)}$ B) $e^{ax}/f(D+a)$

C) $\frac{e^{ax}}{f(-a)^2}$ D) None of these **(04 Marks)**

b. Solve $6\frac{d^2y}{dx^2} + 17\frac{dy}{dx} + 12y = e^{-x}$. **(04 Marks)**

c. Solve the equation $(D^3 - 1)\, y = 3\, \text{Cos}\, 2x$. **(06 Marks)**

d. Solve by the method of undetermined coefficients $(D^2 - 4D + 4)\, y = e^x$. **(06 Marks)**

6 a. Select the correct answer in each of the following :

i) The Wronskian of differential equation $f(D)y = \phi\,(x)$ is

A) $W = y_1y_2^1 - y_2y_1^1$ B) $W = y_2\, y_1$

C) $W = y_2^2$ D) $y_1 + y_2$

ii) To transform $(ax + b)^2\, y'' + (ax + b)\, y' + y = \phi\,(x)$ into a linear differential equation with constant coefficients put t =

A) Log (ax + b) B) e^x C) x D) e^{2x}

iii) The solution of the differential equation $y'' + 4y' + 4y = 0$, satisfying the conditions $y(0) = 1$ and $y\,(1) = 1$ is

A) $(c_1 + c_2x)^{-2x}$ B) Cos x + Sin x C) 2 Sinx D) Sinx Cosx

iv) $c_1e^x + c_2e^{-x}$ is the general solution of

A) $(D^2 + 1)\, y = 0$ B) $(D^2 - 1)\, y = 0$ C) $(D + 1)\, y = 0$ D) None of these **(04 Marks)**

b. Solve $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = (1 + x)^2$. **(04 Marks)**

c. Solve by the method of variation of parameters $(D^2 + a^2)\, y = \sec ax$. **(06 Marks)**

d. Solve the initial value problem $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$ given that $x\,(0) = 0$, $\frac{dx}{dt}(0) = 15$. **(06 Marks)**

7 a. Select the correct answer in each of the following :

i) Laplace transform of cosat is

A) $\frac{s}{s^2+a^2}$ B) $\frac{a}{s^2-a^2}$ C) $\frac{1}{s^2+a^2}$ D) $\frac{1}{s-a}$

ii) Laplace transform of sinhat is

A) $\frac{a}{s^2+a^2}$ B) $\frac{a}{s^2-a^2}$ C) $\frac{s}{s^2+a^2}$ D) $\frac{1}{s^2+a^2}$

iii) Laplace transform of $[e^{at} f(t)]$ is

A) $\bar{f}(s+a)$ B) $\bar{f}(s-a)$ C) $\bar{f}(s)$ D) None of these

iv) Laplace transform of (t^5) is equal to

A) $\frac{120}{s^6}$ B) $\frac{10}{s^3}$ C) $\frac{125}{s^6}$ D) $\frac{122}{s^4}$. **(04 Marks)**

b. Find the Laplace transform of e^{3t} Sin5t Sin3t. **(04 Marks)**

c. Given $f(t)=\begin{cases} E, & 0<t<a/2 \\ -E, & a/2<t<a \end{cases}$, where f(t + a) = f(t), show that $L\{f(t)\}=\frac{E}{S}\tanh\left(\frac{as}{4}\right)$. **(06 Marks)**

d. Find the Laplace transform of the function $[e^{t-1} + \text{Sin}(t-1)]\, u(t-1)$. **(06 Marks)**

8 a. Select the correct answer in each of the following :

i) Inverse Laplace transform of $\frac{1}{s+1}$ is

A) e^{at} B) e^{-t} C) e^{2t} D) t

ii) Inverse Laplace transform of $\frac{s+5}{s^2-6s+13}$ is

A) e^{3t} (Cos2t + 4 Sin2t) B) Cos 2t

C) e^{t} D) None of these

iii) Inverse Laplace transform of $\frac{1}{s^4}$ is

A) $\frac{t^2}{2!}$ B) $t^3/3!$ C) $t^4/4!$ D) $t^2/4!$

iv) $L^{-1}\left(\frac{1}{s^2+5}\right)=$

A) $\frac{1}{\sqrt{5}}\text{Sin}(\sqrt{5}t)$ B) $\frac{1}{\sqrt{6}}\text{Cos}\sqrt{6}t$ C) $\frac{1}{\sqrt{7}}\text{Sin}\sqrt{6}t$ D) None of these. **(04 Marks)**

b. Find the inverse Laplace transform of $\frac{s^2-2s^2+1}{s^5}$. **(04 Marks)**

c. Find the inverse Laplace transform of $\frac{1}{s(s+1)(s+2)(s+3)}$. **(06 Marks)**

d. Solve by using Laplace transforms $\frac{d^2y}{dt^2}+k^2y=0$ given that y (0) = 2, y′(0) = 0. **(06 Marks)**

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